1.9 – Compositions of Matrix Transformations

The **composition of $\boldsymbol{T_B}$ with $\boldsymbol{T_A}$** is achieved by first applying the matrix transformation T_A to a vector and then applying the matrix transformation T_B to the image vector. We denote the composition of T_B with T_A by $T_B \circ T_A$ which is read "T_B circle T_A." This is also expressed as $(T_B \circ T_A)(\mathbf{x}) = T_B\big(T_A(\mathbf{x})\big)$.

Theorem 1.9.1 If $T_A: R^n \to R^k$ and $T_B: R^k \to R^m$ are matrix transformations, then $T_B \circ T_A$ is also a matrix transformation, and $T_B \circ T_A = T_{BA}$.

8. Find the standard matrix for the stated composition in R^2.

a. A rotation about the origin of $60°$, followed by an orthogonal projection onto the x-axis, followed by a reflection about the line $y = x$.

b. An orthogonal projection onto the x-axis, followed by a rotation about the origin of $45°$, followed by a reflection about the y-axis.

c. A rotation about the origin of $15°$, followed by a rotation about the origin of $105°$, followed by a rotation about the origin of $60°$.

12. Let $T_1(x_1, x_2, x_3) = (4x_1, -2x_1 + x_2, -x_1 - 3x_2)$ and $T_2(x_1, x_2, x_3) = (x_1 + 2x_2, -x_3, 4x_1 - x_3)$.

a. Find the standard matrices for T_1 and T_2.

b. Find the standard matrices for $T_2 \circ T_1$ and $T_1 \circ T_2$.

c. Use the matrices obtained in part (b) to find formulas for $T_1\big(T_2(x_1, x_2, x_3)\big)$ and $T_2\big(T_1(x_1, x_2, x_3)\big)$.

If $T_A: R^n \to R^n$ is a matrix operator whose standard matrix A is invertible, then T_A is **invertible**, and the **inverse** of T_A is $T_A^{-1} = T_{A^{-1}}$.

20. Determine whether the matrix operator $T: R^3 \to R^3$ defined by the equations is invertible; if so, find the standard matrix for the inverse operator, and find $T^{-1}(w_1, w_2, w_3)$.

a.

$$w_1 = x_1 - 2x_2 + 2x_3$$
$$w_2 = 2x_1 + x_2 + x_3$$
$$w_3 = x_1 + x_2$$

b.

$$w_1 = x_1 - 3x_2 + 4x_3$$
$$w_2 = -x_1 + x_2 + x_3$$
$$w_3 = -2x_2 + 5x_3$$